5 galline e 3 galli

Prob. di riuscire a uscire dal pollaio

è $p = 0.20$

X = n° di galline che escono

Y = n° di animali che escono

(a) $X \sim ?$ (b) $Y \sim ?$

(c) Calc. la prob. che esca solo qualche gallo

(d) Sapendo che sono uscite non più di 2 galline, calc. la prob. che siano usciti esatt 5 animali

$$X \sim B(5, p)$$

$$X_i = \begin{cases} 1 & \text{se l'i-esima gallina esce} \\ 0 & \text{~~se~~ no} \end{cases}$$

$$\underbrace{X = \sum_{i=1}^{5} X_i}\ ; \quad X_i \ \text{indipendenti} \sim B(1, p)$$

$$\left.\begin{array}{l} X \sim B(m, \underline{p}) \\ Z \sim B(n, \underline{p}) \end{array}\right\} \; \text{indipend}$$

$$X + Z = Y \sim B(m+n, p)$$

$$Y = \left(\underbrace{\sum_{i=1}^{5} X_i} + \underbrace{\sum_{i=1}^{3} Z_i} \right) = \sum_{i=1}^{8} Y_i$$

$$Z_i = \begin{cases} 1 & \text{se l'i-esimo giallo esce} \\ 0 & \text{sennò} \end{cases}$$

$$i = 1, 2, 3$$

$$Y \sim B(8, p) \qquad X_1, X_2, X_3, X_4, X_5, Z_1, Z_2, Z_3$$

- X = n° di galline che escono
- Y = n° di animali che escono

$$P(X=0, Y \geq 1)$$

$$Y = X + Z$$

Z ↑ n° di galli che escono

X e Z indipendenti!

$$\{X=0,\ \underline{Y\geq 1}\} \subseteq \{Z\geq 1\}$$

$$= \{X=0,\ Z\geq 1\}$$

$$= \{X=0,\ \cancel{X}+Z\geq 1\}$$

$$P(X=0,\ Z\geq 1) = P(X\in A,\ Y\in B)$$

$$A=\{0\} \qquad B=[1,+\infty)$$

$$P(Y=5 \mid X \le 2) = \frac{P(Y=5,\ X \le 2)}{P(X \le 2)} =$$

$$P(X \le 2) = \sum_{x=0}^{2} P(X=x) = \sum_{x=0}^{2} \binom{5}{x} p^{x} (1-p)^{5-x}$$

$$P(Y=5,\ X\le 2) =$$

$$P(X+Z=5,\ X\le 2)$$

$$\{X+Z=5,\ X\le 2\} \subset$$

$$\subset \{Z\ge 3,\ X\le 2\}$$

$$\frac{P(X+Z=5,\ X\le 2)}{P((X,Z)\in S)} = \sum_{(x,z)\in S} p(x,z) = \sum_{(x,z)\in S} p_X(x)\, p_Z(z)$$

$X \sim B(5,p)$

$Z \sim B(3,p)$

$S = \{(x,z)\in \mathbb{N}^2 : x+z=5,\ x\le 2\}$

$= \{(0,5),\ (1,4),\ (2,3)\}$

$= \{$

$$\underbrace{p_X(0)\, p_Z(5)}_{=0} + \underbrace{p_X(1)\, p_Z(4)}_{=0} + p_X(2)\, p_Z(3) =$$

Siano X e Y 2 v. a. indipend. entrambe di densità geometrica di par. p (dato)

(a) Calc. $P(X+Y=5)$

(b) Calc. la densità di $X+Y$

(c) Calc. $P(X=k \mid X+Y=5)$

$$\{X+Y=5\} = \{X=1, Y=4\} \cup \{X=2, Y=3\} \cup \{X=3, Y=2\} \cup \{X=4, Y=1\}$$

$$p_X(k) = P(X=k) = p(1-p)^{k-1}$$

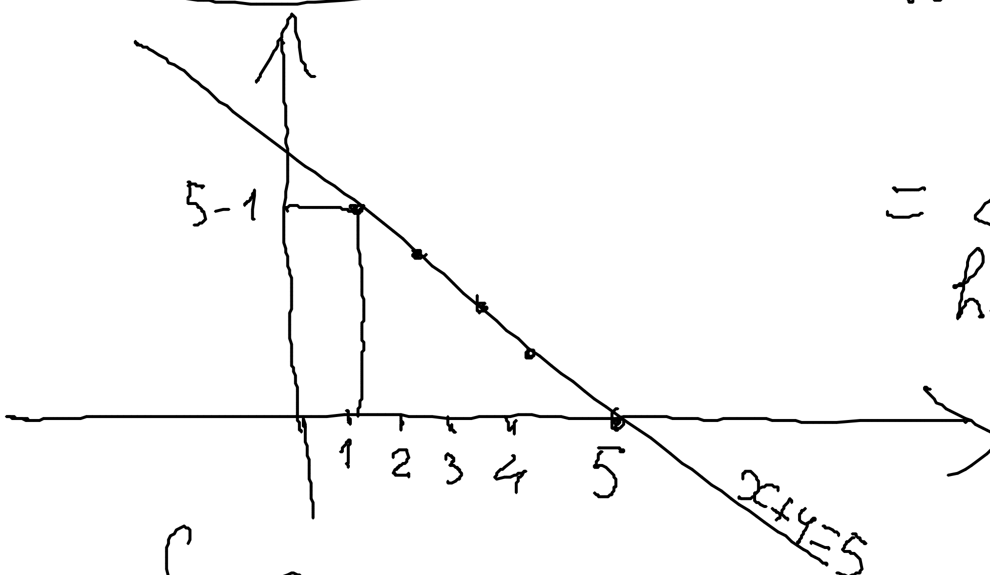
$$k = 1, 2, \ldots$$

$$P(X+Y=5) = \sum_{i=1}^{4} P(X=i, Y=5-i)$$

$$= \sum_{i=1}^{4} p_X(i)\, p_Y(5-i)$$

$$P(X+Y=5) = P((X,Y)\in S).$$

$$= \boxed{\sum_{(h,k)\in S} p(h,k)} = \sum_{h=1}^{4} p_X(h)\, p_Y(5-h)$$



$$= \sum_{h=1}^{4} p(1-p)^{h-1}\, p(1-p)^{5-h-1}$$

$$= \boxed{4p^2(1-p)^3}$$

$$S = \{(x,y) : x+y=5\}$$

$$P(X+Y=r)= \qquad r=2,3,4,5,\dots$$

$$=\sum_{(h,k)\in S} p_X(h)\, p_Y(k) = \qquad h+k=r$$

$$\sum_{h=1}^{r-1} \underline{p_X(h)}\, p_Y(r-h) = (r-1)p^2 \underline{(1-p)^{r-2}}$$

$$\sum_{k=1}^{\infty} p_X(k)\, p_Y(n-k) =$$

$$= \sum_{k=1}^{\infty} p\,(1-p)^{k-1}\, p\,(1-p)^{n-k-1} =$$

$$= p^2\,(1-p)^2 \sum_{k=1}^{n} 1$$

$$P(X=k \mid X+Y=5) =$$

$$= \frac{P(X=k,\ X+Y=5)}{P(X+Y=5) = 4p^2(1-p)^3}$$

$$P(X=k,\ X+Y=5) = P(X=k,\ Y=5-k) =$$

$$= P(X=k)\ P(Y=5-k) = p(1-p)^{k-1}\ p(1-p)^{5-k-1} =$$

for $k=1,2,3,4$

$$= p^2(1-p)^3$$

$$k \longmapsto P(X = k \mid X+Y = 5) =$$

$$= \begin{cases} \dfrac{1}{4} & k = 1, 2, 3, 4 \\ 0 & \text{altrove} \end{cases}$$

è una densità (dens. condizionale di X, dato $\{X+Y=5\}$)

X, Y indipend

$$\left.\begin{array}{l} X \sim \Pi_\lambda \\ Y \sim \Pi_\mu \end{array}\right\} \text{ indipendenti}$$

Calc. la legge di $X+Y$

$$= \underbrace{P(X=0)}_{\binom{5}{0} p^0 (1-p)^{5-0}} P(Z \geq 1)$$

$$\left(1 - P(Z=0)\right)$$

$$= \left(1 - \binom{3}{0} p^0 (1-p)^{3-0}\right)$$

$$P(X+Y=r)= \qquad r \text{ intero} \quad r=0,1,2,3\ldots$$

$$=\sum_{h} P(X=h)\,P(Y=r-h)$$

$$=\sum_{h} p_X(h)\;p_Y(r-h)=$$

$$=\sum_{h=0}^{r} \frac{\lambda^h e^{-\lambda}}{h!}\,\frac{\mu^{r-h}}{(r-h)!}\,e^{-\mu}=$$

$$= \sum_{h=0}^{r} \frac{\lambda^{h}}{h!} e^{-\lambda} \frac{\mu^{r-h}}{(r-h)!} e^{-\mu} =$$

$$= \frac{e^{-(\lambda+\mu)}}{r!} \sum_{h=0}^{r} \frac{r!}{h!\,(r-h)!} \lambda^{h} \mu^{r-h} =$$

$$= \frac{e^{-(\lambda+\mu)}}{r!} \sum_{h=0}^{r} \binom{r}{h} \lambda^{h} \mu^{r-h} =$$

$$P(X+Y=r) = \begin{cases} \dfrac{(\lambda+\mu)^r}{r!} e^{-(\lambda+\mu)} & r = 0, 1, 2 \ldots \\ 0 & \text{elsewhere} \end{cases}$$

$$X+Y \sim \Pi_{\lambda+\mu}$$